

Parabolically induced mod p representations of reductive p-adic

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$$F/\mathbb{Q}_p < \infty$$

$$\mathbb{F}_p((t))$$

\underline{G} reductive connected group over F

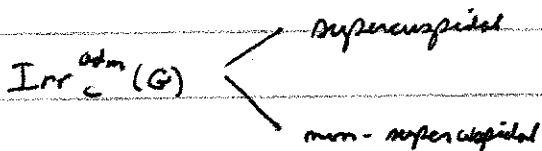
$$G = G(F), \quad T \subset B = Z \cup$$

min. parabolic

C alg.-closed field of char. p .

Representations admissible τ_K of G .

(τ_K^n has fin. dim. for $H \subset G$ open compact.)



Classify the non-supercuspidal inv. adm. reps.

Barthel-Lima: $GL(2, F)$

Merzjag: $GL(n, F)$ all n , $F \supset \mathbb{Q}_p$

Abe: split $F \supset \mathbb{Q}_p$

A.H.H.V: general G and F

$$P \supset B$$

$$\parallel$$

$$MN$$

$$\cup$$

$$Z$$

$$\text{Ind}_P^G: \text{Mod}_C M \longrightarrow \text{Mod}_C(G)$$

$\underbrace{\quad}_{\text{rep. of } M/C}$

$$(\tau, W)$$

$\text{Ind}_P^G(W) \supset G$ right translation

$$= \{ f: G \rightarrow W: \}$$

$$f(nmgk) = \tau(m) f(g) \}$$

K - open compact $\subset H$

Prop.: Parabolic induction is faithful, with a left and right adjoint, respects admissibility.

Def: $\tau \in \text{Irr}^{\text{adm}}(G)$ is supercuspidal if it is not a subquotient of $\text{Ind}_P^G \tau$ for $P \neq G$, $\tau \in \text{Irr}^{\text{adm}}(M)$.

Supercuspidal admissible triple: (P, σ, Q) $B \subset P = MN$
 $\sigma \in \text{Irr}_c^{\text{adm}}(\tau)$ supercuspidal, $P \subset Q \subset P(\sigma)$ where $P(\sigma)$ is the largest group where σ can be extended trivially on N .

Generalized Steinberg representation: $\text{St}_Q^{P(\sigma)}(\sigma) :=$

$$\frac{\text{Ind}_Q^{P(\sigma)} e(\sigma)}{\sum_{Q' \leq Q \text{ s.t. } P(\sigma)} e(\sigma)}$$

$$I(P, \sigma, Q) = \text{Ind}_{P(\sigma)}^G \text{St}_Q^{P(\sigma)}(\sigma).$$

Thm: There is a bijection

$$\begin{aligned} (P, \sigma, Q) &\longrightarrow I(P, \sigma, Q) \\ \left\{ \text{supercusp. adm. triple} \right\} &\longrightarrow \text{Irr}_c^{\text{adm}}(G). \end{aligned}$$

First applications ① $\text{Ind}_P^G \tau$ $\tau \in \text{Irr}_c^{\text{adm}}(M)$

finite length, invd subgts are adm, mult. 1

② supercuspidal support

Second applications:

Thm: supercuspidal = supersingular.

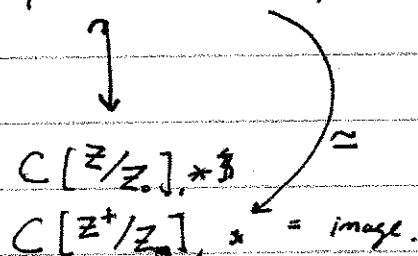
Cartan homomorphism:

K open compact subgroup, "special parahoric" $G = BK$

$B \cap K = Z_0 \cdot U_0 \quad P \cap K = M_0 \cdot N_0$

$C[K \backslash G / K]$, * convolution product

spherical Hecke algebra



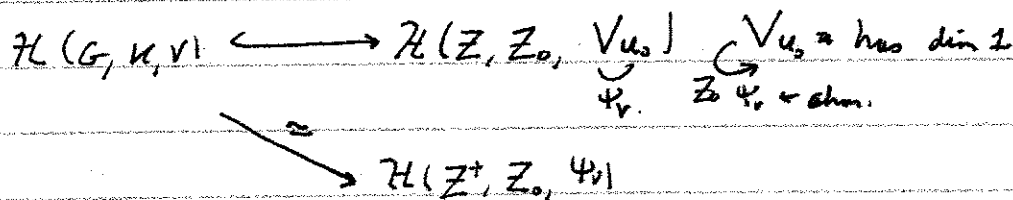
$V \in \text{Irr}_c(K)$ Compact induction $\text{ind}_K^G V$

$= \{ f: G \rightarrow V \text{ compact support} \\ f(kg) = \kappa f(g) \}$

Generalization of spherical Hecke algebra:

$\text{End}_{C_G}(\text{ind}_K^G V) = \mathcal{H}(G, K, V)$

$\{ f: G \rightarrow \text{End}_C(V), f(k_1 g k_2) = \kappa_1 f(g) \kappa_2 \}$
compact support



Z/Z_0 is commutative (Masieles)

Z/Z_0 not comm. in general
proper Sylow

center $Z(Z^+, Z_0, \psi_V)$.

$$\chi \otimes \text{ind}_K^G V$$

$Z(Z^+, Z_0, \psi_V)$

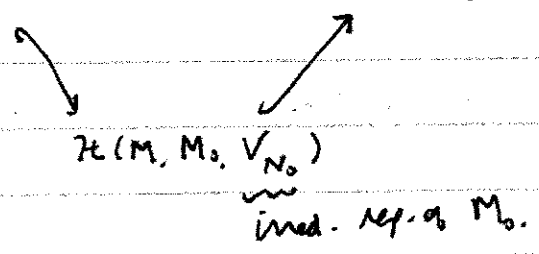
$$\chi: Z(Z^+, Z_0, \psi_V) \rightarrow \mathbb{C}$$

character

$$\pi \in \text{Irr}_C^{\text{adm}}(G)$$

Def: V is a weight of π , χ is an eigenvalue of $Z(G, K, \psi_V)$ in π if π is a quotient of $\chi \otimes \text{ind}_K^G V$

$$Z(G, K, \psi_V) \hookrightarrow Z(Z, Z_0, \psi_{V_0})$$



χ is called supersingular if it does not extend to $Z(M, M_0, \psi_{N_0})$ for all $P = NN_0 \neq G$

π is supersingular if for all weights V in π , all eigenvalues χ in π of χ is supersingular

Going from compact induction to parabolic induction

$$C[K \backslash G] \hookrightarrow \text{Ind}_B^G C[Z_0 \backslash Z] \text{ equivalent to } Z_0(G, K, \psi_V)$$

$$\text{ind}_K^G V \xleftrightarrow{\quad} \text{Ind}_B^G \text{ind}_{Z_0}^Z V_{\psi} \\ \searrow \\ V \text{ "regular"} \quad \text{Ind}_p^G \text{ind}_{M_0}^M V_{N_0} \quad \chi \in \widehat{Z}(M, N_0, V_{N_0}).$$

Restriction = char of $Z(G, \chi, V)$

$$\chi \otimes \text{ind}_K^G V \simeq \chi \otimes \text{Ind}_p^G \text{ind}_{N_0}^N V_{N_0}$$

change of weight \swarrow under some condition

$$\chi \otimes \text{ind}_K^G V \simeq \chi \otimes \text{ind}_K^G V' \quad \psi_V = \psi_{V'}$$

$$Z(G, \chi, V) \simeq Z(Z^+, Z_0, \psi)$$

proved by using the action of the pro-p-Iwahori

Compare $\text{ind}_K^G V$ and $\text{ind}_K^G V'$ inside of

$$\mathcal{H}_c(G, I_u) \quad I_u = \text{pro-}p\text{-radical of the division } I \\ I_u \cap B_0 = Z_u U_0$$

$$\text{Mod}_c(G) \xrightarrow{I_u\text{-inv}} \text{Mod}_{\mathcal{H}_c}(G, Z_u)$$

classification of

left adjoint - $\otimes C[I_u \setminus G]$

$$I_{\mathcal{H}_c(G, Z_u)}$$

$$V \in \text{Rad}_c G \Rightarrow V^{I_u} \neq 0$$

$\neq 0$

super-singular

non-singular

~~XXXXXXXXXXXXXXXXXXXX~~

$$\begin{array}{ccc}
 \text{Mod}_C(G) & \begin{array}{c} \xrightarrow{\text{In-Inv}} \\ \xleftarrow{\quad} \end{array} & \text{Mod}_{\mathbb{Z}_2}(G, \text{Inv}) \\
 \uparrow \text{Ind}_P^G & & \uparrow \text{Ind}_P^G \\
 \text{Mod}_C(M) & \begin{array}{c} \xrightarrow{? \text{-inv}} \\ \xleftarrow{\quad} \end{array} & \text{Mod}_{\mathbb{Z}_2}(M, \text{Inv}, \mathbb{Z})
 \end{array}$$

Thm: 1) Parabolic induction commutes with pro- p -decomposition function and its left adjoint.

2) $\pi \in \text{Irr}_C^{\text{adm}}(G)$ supersingular $\iff \pi^{\text{In}}$ has all its irreducible subquotients supersingular.