MATH 1080 — FINAL EXAM

December 8, 2014

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 11 pages including this cover. There are 6 problems.
3. Write your name on the top of EVERY sheet of the exam at the START of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. If you use an integral in an applied problem, you must explain via slicing how you derived the integral.
8. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.
9. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	30	
2	10	
3	25	
4	10	
5	15	
6	15	
TOTAL	105	

Name: ____

1. (5 points each) Evaluate the following.

(a) $\int_0^3 3ue^{u^2}\,du$

(b) $\sum_{j=0}^{\infty} \frac{(\ln 2)^j}{j!}$

Name: ____

(c) $\int \frac{1}{(x^2+4)(x-2)}dx$

(d) $\lim_{n\to\infty} \sum_{j=0}^{n-1} \cos\left(\frac{j}{n}\right)\frac{1}{n}$

Name: ____

(e) The area enclosed by the lemniscate $r^2 = 4\cos(2\theta)$.

(f) $\displaystyle\int_1^3 \frac{1}{(x-2)^2}\,dx.$

Name: ____

2. (10 points) Find the Taylor series centered at 0 for $f(x) = \dfrac{3}{2 - x^2}$. Determine the values of x for which $f(x)$ equals its Taylor series.

Name: ____

3. (10+10+5 points) According to Einstein's special theory of relativity, a body of mass m_0 at rest has rest energy $E_0 = m_0 c^2$ due to the mass itself where c is the speed of light. The same body, moving at speed v, has total energy $E(v) = \dfrac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$. Note $E(0) = E_0$. The kinetic energy $K(v)$ is the difference between the total energy and the rest energy: $K(v) = E(v) - E(0)$.

(a) Give the first two non-zero terms in the Taylor expansion of $K(v)$ centered at 0. Explain how this recovers the classical formula for kinetic energy $K_{\text{classical}}(v) = \frac{1}{2} m_0 v^2$.

Name: ______________________________

(b) If one only considers velocities v that satisfy $0 \leq v \leq \frac{c}{2}$, what is the maximum error in using $K_{\text{classical}}(v)$ in place of $K(v)$?

Name: ____

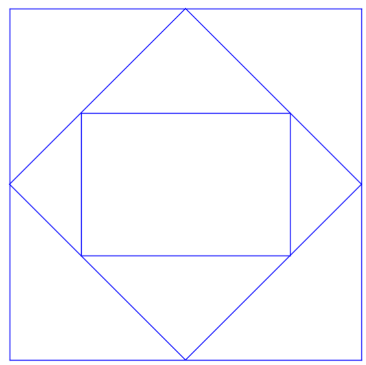
(c) If $v = 0.01c$, by what percentage does the approximation $K(v) \approx K_{\text{classical}}(v)$ change if includes the first two nonzero terms to approximate $K(v)$?

Name: ____

4. (10 points) Coulomb's law states that the force exerted on two point charges q_1 and q_2 separated by a distance of r is given by $F(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ where ε_0 is a constant known as the permittivity of free space. Suppose the electric charge q_1 is concentrated at the origin of the coordinate line and that it repulses a like charge q_2 from the point $x = a$ to the point $x = b$. Derive a formula for the total work done by the repulsive force in moving the charge q_2 from $x = a$ to $x = b$.

Name: ____

5. (15 points) An infinite sequence of nested squares is constructed as follows: Starting with a square of side length 2, each square in the sequence is constructed from the preceding square by drawing line segments connecting the midpoints of the sides of the square. Find the sum of the areas of all the squares in the sequence. The first three squares are pictured here.



Name: ____

6. (15 points) One learns in calculus that a function $y = f(x)$ that is differentiable at $x = a$ can be approximated near $x = a$ by the tangent line, namely, $f(x) \approx f(a) + f'(a)(x - a)$ for $x \approx a$. Explain (using example, pictures, etc.) to someone that has had Math 1060 what a Taylor polynomial is and what it can be used for. It would be helpful to show them where the formula comes from and not just state it.