MATH 116 — FIRST MIDTERM EXAM

Fall 2003

NAME: ____________________________ ID NUMBER: ___________________________

INSTRUCTOR: ____________________ SECTION NO: ___________________________

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	7	
2	10	
3	6	
4	10	
5	10	
6	10	
7	10	
8	10	
9	12	
10	15	
TOTAL	100	

1. (7 points) The *sine-integral* function $Si(x)$ is defined by

$$Si(x) = \int_0^x \frac{\sin t}{t}\, dt.$$

What is the derivative of $Si(x^3)$?

Answer: $\frac{d}{dx} Si(x^3) =$ ______________________

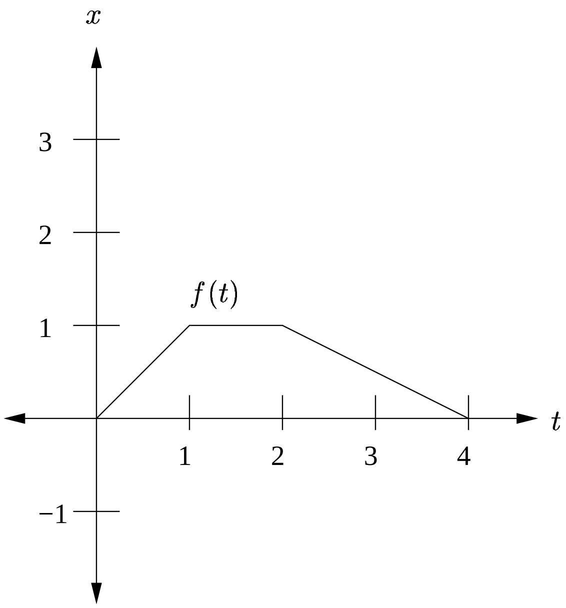
2. (10 points) Let $g(x)$ be a continuously differentiable functions of x that satisfies $g(1) = 2$, $g(5) = 6$, and $\int_1^5 g(x)\, dx = -2$. Compute, showing all your work,

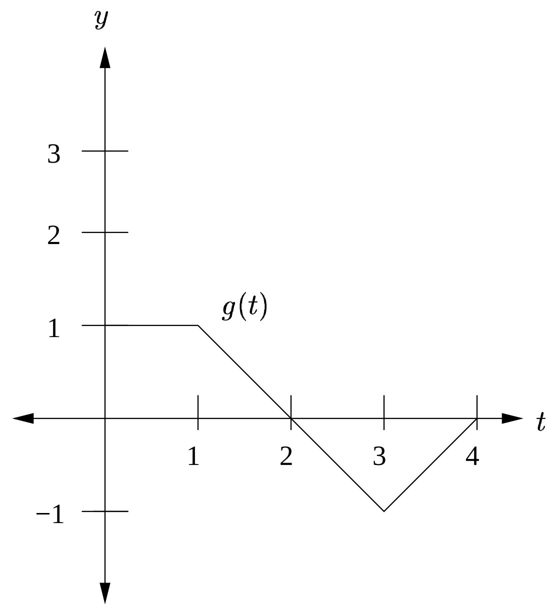
(a) $\int_1^5 xg'(x)dx =$ ____________ .

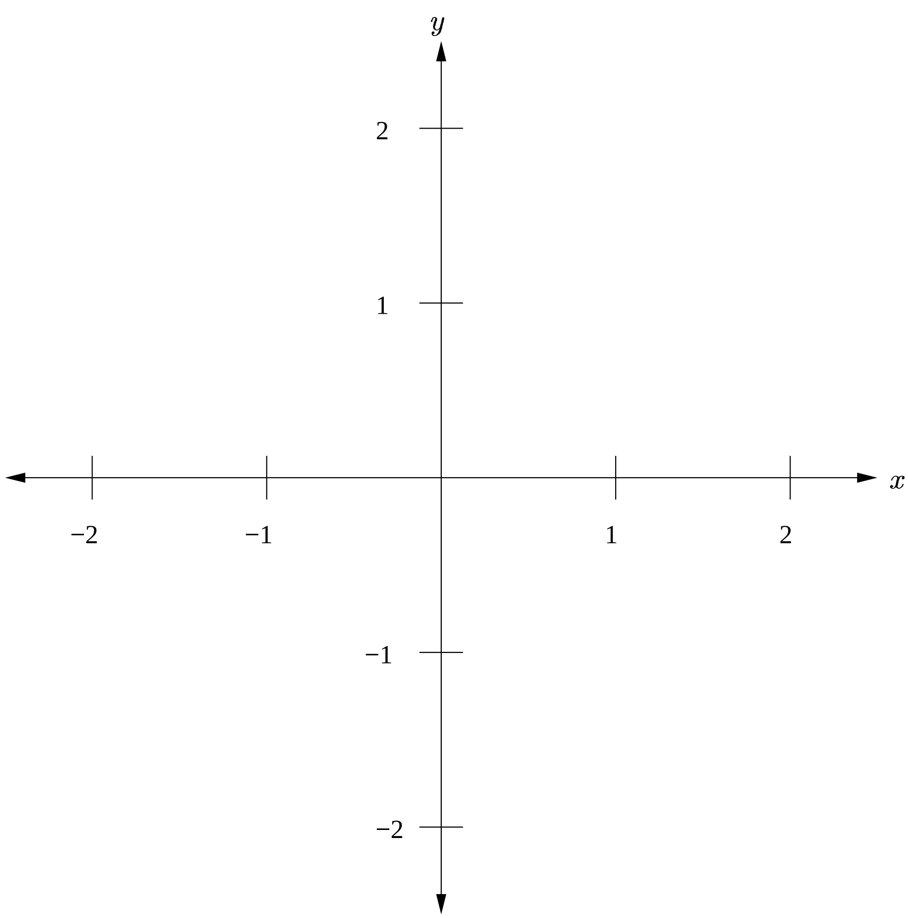
(b) $\int_2^3 g(4x - 7)\, dx =$ ____________

3. (6 points) Let $r(t)$ represent the rate that the height of a child changes per year (in inches per year), where $t = 0$ corresponds to the birth date of the child. Explain the meaning of the quantity $\int_4^8 r(t)\, dt$. (Remember to use units.)

4. (10 points) Let f, g be the functions with graphs as shown in the first two of the figures below. On the axes given below the graphs of f and g, sketch a graph of the curve with parametric equations $x = f(t)$, $y = g(t)$, $0 \leq t \leq 4$.







5. (10 points) Circle the correct answer(s) to each of the following questions or circle True or False, as appropriate. (No explanations necessary.) In all the questions, f is a continuous function defined on an interval $a \le x \le b$.

(a) If f is a decreasing function, then for every $n = 1, 2, 3, \ldots$, the approximation to $\int_a^b f(x)\,dx$ given by LEFT(n) is an

underestimate. overestimate. could be either.

(b) If f is concave down, then for every $n = 1, 2, 3, \ldots$, the approximation to $\int_a^b f(x)\,dx$ given by TRAP(n) is an

underestimate. overestimate. could be either.

(c) If n is very large, then the midpoint rule MID(n) always gives one the exact value of $\int_a^b f(x)\,dx$.

True. False.

(d) The approximation to $\int_a^b f(x)\,dx$ given by the trapezoidal rule TRAP(n) is always more accurate than that given by the left rule, LEFT(n).

True. False.

(e) Given a graph of $f'(x)$, one can uniquely determine the graph of $f(x)$.

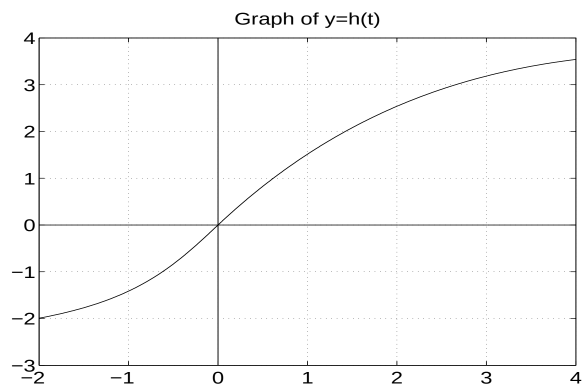
True. False.

6. (10 pts.) A leak is found in the dam your company just finished constructing. The reservoir behind the dam contains 10 million gallons of water when the leak is first discovered, and it is believed that the water is leaking out at a rate of $r(t) = 0.23e^{\frac{t}{1+t}}$ millions of gallons per hour t hours after this time. If it takes your crew 5 hours to repair the leak, how much water would be lost? (Be sure to show your work and explain how arrived at your answer.)

7. (10 points) A function F is defined for $-2 \leq x \leq 4$ by the formula

$$F(x) = \int_0^x e^{-h(t)}\, dt$$

where h is the function with the graph shown below.



(a) True or False? For $-2 < x < 4$, $\quad F'(x) = e^{-h(t)}$. T F

(b) On which interval or subintervals of $[-2, 4]$ is F increasing? decreasing?

(c) On which interval or subintervals of $[-2, 4]$ is F concave up? concave down?

8. (10 points) Let f be a continuous, positive function for $x \geq 1$.

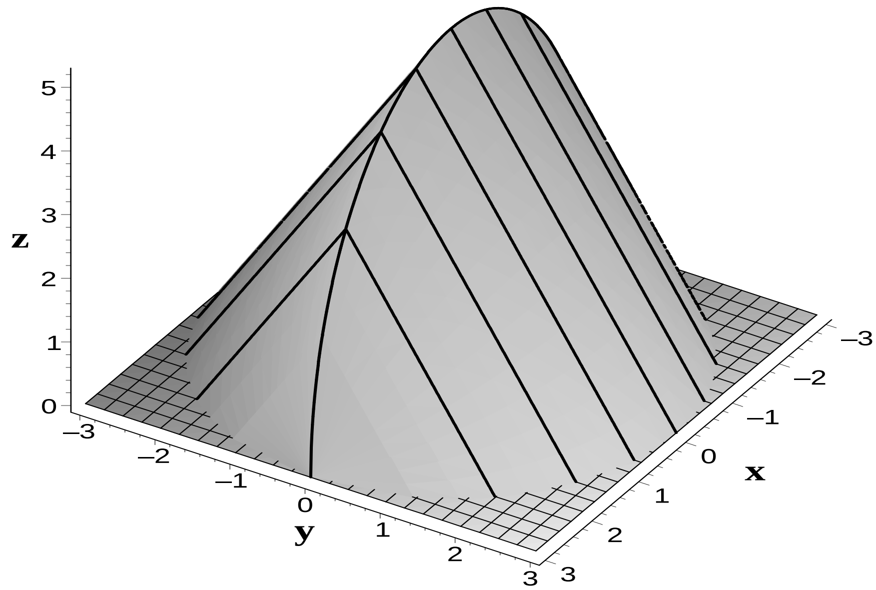
(a) Define what it means to say that $\int_1^\infty f(x)\,dx$ converges.

(b) If f from part (a) is such that $\int_1^\infty f(x)\,dx$ converges and if g is another continuous positive function for $x \geq 1$ that satisfies

$$g(x) \leq 5f(x) + \frac{3}{x^2}$$

then is it necessarily true that $\int_1^\infty g(x)\,dx$ converges? (Explain why or why not).

9. (12 points) It's a beautiful sunny day and you are at the beach. You manage to build the most spectacular sand castle ever. Unforunately, fate is cruel and a rogue wave hits the beach and washes over your sandcastle. But, fate also has a kinder side and it leaves you a shapely mound of sand as pictured below. The mound has as a base the interior of the circle $x^2 + y^2 = 9$ in the x-y plane and has cross sections by planes perpendicular to the x-axis given by equilateral triangles with one side in the x-y plane.



(a) Sketch and label the dimensions of a typical slice of the sand mound perpendicular to the x-axis for $-3 < x < 3$. What is the volume of this slice in terms of x?

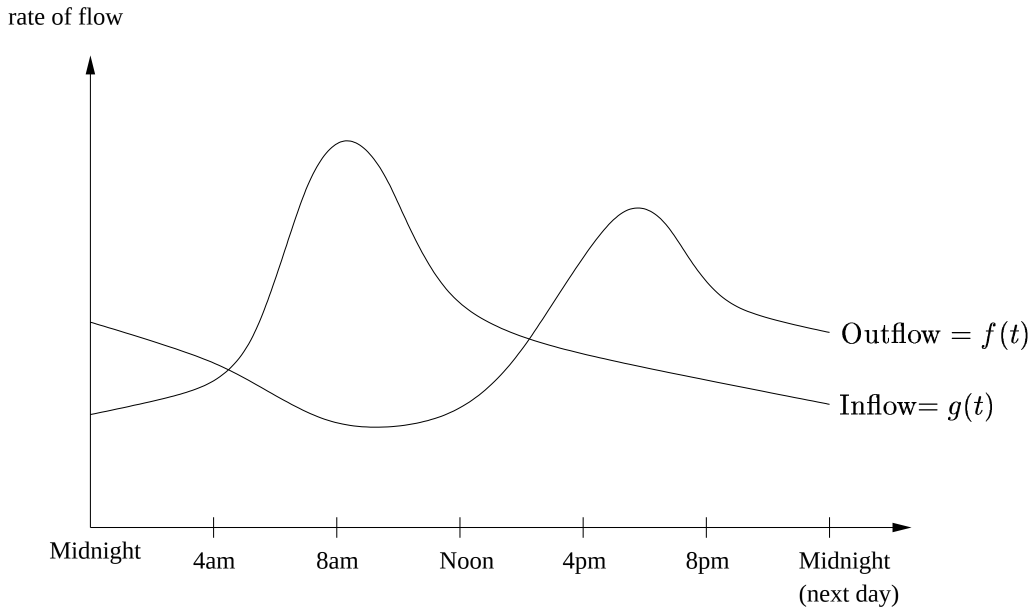
Problem continued on next page.

Continuation of problem from the previous page.

(b) Write a Riemann sum and then a definite integral representing the volume of the sand pile.

(c) Find the exact volume of the solid. If you can't compute the volume exactly, give the most accurate approximation you can and explain how you found it.

10. (15 points) The graphs shown represent the flow of traffic (in number of motor vehicles per minute) in and out of Ann Arbor on a typical weekday.



(a) During the course of the day, at what time is the largest number of cars in Ann Arbor? Give an explanation of how you arrived at this answer.

(b) At what time is the number of cars in Ann Arbor increasing the most rapidly? Decreasing the most rapidly? Again, please give an explanation of how you arrived at this answer.

Problem continued on next page.

Continuation of problem from the previous page.

(c) Sketch possible graphs of the inflow of traffic and the outflow of traffic in for Ann Arbor on a football Saturday if we assume kickoff is at 3pm. Explain how you arrived at the graph drawn.