MATH 116 — SECOND MIDTERM EXAM

November 11, 2003

NAME: ____

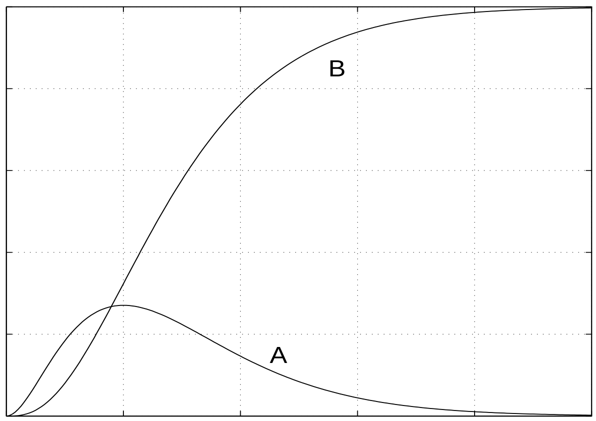
INSTRUCTOR: ____________________ SECTION NO: __________________________

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	10	
2	8	
3	10	
4	15	
5	5	
6	10	
7	14	
8	12	
9	10	
10	6	
TOTAL	100	

1. (10 points) The figure shows the graphs of two functions, A and B, one of which is a probability density function and the other of which is the corresponding cumulative distribution function.
(a) Which curve represents the density function and which represents the cumulative distribution function?

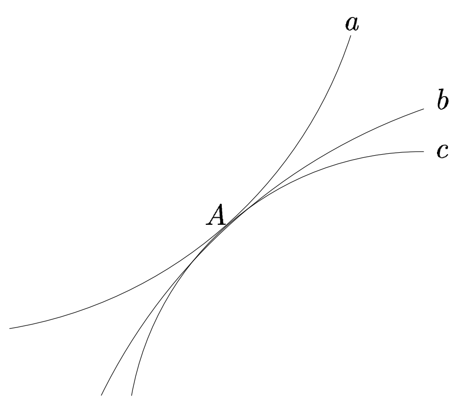
(b) Put reasonable values on the tick marks on each of the axes.



2. (8 points) The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{3^n x^{2n}}{n+1}$ is $R =$ ______________.
(Show your work and/or explain your reasoning.)

3. (10 points) Three functions f_1, f_2, and f_3, have graphs that pass through a point A and are shown in the figure. Second degree Taylor polynomials for these functions are as follows:

$$\begin{aligned} f_1(x) &\approx 10 + (x-5) - (x-5)^2 \\ f_2(x) &\approx 10 + (x-5) + (x-5)^2 \\ f_3(x) &\approx 10 + (x-5) - 5(x-5)^2 \end{aligned}$$



(a) What are the coordinates of the point A?

(b) Which function goes with which graph? Explain how can you tell?

4. (15 points) Circle "True" or "False" for each of the following statements. Circle "True" only if the statement is always true. No explanation is necessary.

(a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges.

True. False.

(b) If $0 \leq a_n \leq b_n$ for all n, and if $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

True. False.

(c) If $P_4(x) = 5 + 6\,(x-a) + 2\,(x-a)^2 + 37\,(x-a)^3 + 21\,(x-a)^4$ is the 4th degree Taylor polynomial for $f(x)$ about $x = a$, then $f^{(3)}(a) = 37$.

True. False.

(d) If the power series $\sum_{n=0}^{\infty} C_n(x-3)^n$ converges for $x = 1$, then it also converges for $x = 4$.

True. False.

(e) The infinite series $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^5+3}$ converges.

True. False.

5. (5 points) Express the number x whose repeating decimal expansion is 6.17636363636363... as the sum of an infinite series.

6. (10 points) Einstein's special theory of relativity states that an object's length contracts as its velocity increases according to the formula

$$L(v) = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

where L_0 is the length of the object at rest, v is the velocity of the object, and c is the speed of light. (Recall from physics that $v < c$ necessarily)

(a) Approximate $L(v)$ by its second degree Taylor polynomial near $v = 0$.

(b) What is the approximate error in your approximation from part (a) in terms of v when v is small compared to c?

(c) By what percentage will the length of the object contract when it is travelling at a velocity of 99% of the speed of light?

7. (14 points) Suppose the amount of time x that riders wait for the bus to arrive at a certain bus stop is given by the probability density function

$$f(x) = \frac{1}{10}e^{-\frac{1}{10}x}$$

where x is measured in minutes.

(a) What percentage of the time will a rider wait less than 5 minutes for the bus to arrive? (Show your work.)

(b) What is the mean waiting time until the next bus arrives? (Show your work.)

(c) What is the median waiting time until the next bus arrives? (Show your work)

8. (12 points) Beginning in 1749 the Bank of England issued securities known as *British consols* which pay the owner or his heirs a fixed amount of money each year forever. British consols can still be bought and sold in today's securities market.

(a) What is the present value (in British pounds) of a British consol that pays 10 pounds per year? Assume that the first payment is one year from the date of purchase and that the annual interest rate is 5% per year. (Show your work.)

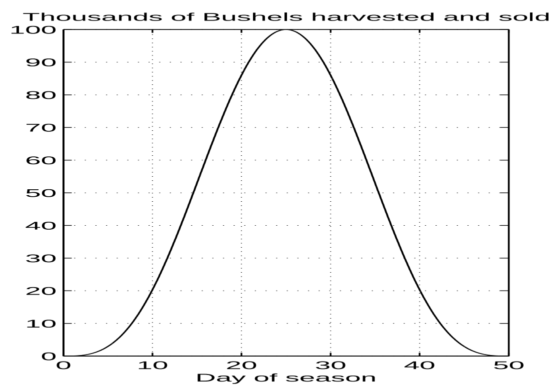
(b) Under the same assumptions as in part (a), what is the present value of the first thirty annual payments? (Show your work.)

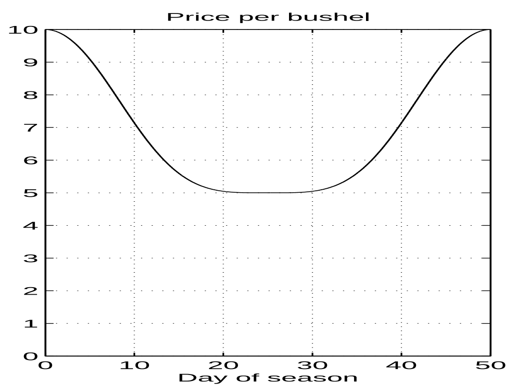
(c) Suppose that, instead of annual payments, the payments are made as a continuous income stream at a constant rate of 10 pounds per year. Assume the interest rate remains at 5% but is now continuously compounded. What then would be the present value of the consol? (Show your work.)

9. (10 points) The finance department of Giant Corporate Farms is forecasting their returns from next season's tomato crop. During the fifty-day harvesting season, they predict being able to harvest $B(t)$ thousand bushels of tomatoes per day and sell them at a price of $P(t)$ dollars per bushel on the t-th day after the beginning of the harvest. They estimate that $B(t)$ and $P(t)$ are given by the following functions whose graphs are shown in the figure.

$B(t) = 100\sin^2(\pi t/50)$ bushels (1,000's) per day $\quad$ $P(t) = 5 + 5\cos^4(\pi t/50)$ dollars per bushel.

The price of tomatoes drops rapidly as the number available for sale increases so while Giant Farms begins the season selling tomatoes for \$10 per bushel, at the height of the harvesting season they receive only \$5 per bushel.





(a) Approximately how much money does Giant Farms predict they will receive on day t of the season?

(b) Set up an integral that is equal to the amount of money that Giant Farms expects to receive in total for the 50 day harvest. Then compute this amount of money by evaluating the integral (any method allowed, including use of your calculator). Be sure to explain how you obtained the answer.

Continued on next page

Continuation of problem 9.

(c) Explain how to calculate the average price per bushel that Giant Farms will receive for their tomatoes.

10. (6 points) Explain how Taylor polynomials can be viewed as generalizations of linear approximation. (A good answer to this problem could begin by discussing a special case such as $P_2(x)$, illustrating your points with graphs and equations.)