MATH 115 — FIRST MIDTERM EXAM

October 13, 2004

NAME: ______________________________

INSTRUCTOR: ____________________ SECTION NO: ____________________

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
TOTAL	100	

1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only is the statement is *always* true. No explanation is necessary.

(a) $\log^{-1}(x) = \frac{1}{e^x}$.

True False

(b) If a function is continuous at a point a, then it must also be differentiable at a.

True False

(c) Suppose f is a continuous function on the interval $[5, 8]$ and that $f(5) = -2$ and $f(8) = 3$. Then f has a zero on the interval $(5, 8)$.

True False

(d) $\lim_{x \to 6} \frac{|x-7|}{x-7}$ exists and equals 1.

True False

(e) Suppose f is a continuous function, $f(1) = 6$, and f is concave up on the interval $(-10, 10)$. It is possible that $f(5) = 4$.

True False

(f) Suppose f is a continuous function, $f(1) = 6$, and $f'(x) > 0$ for all x between 0 and 5. Then it is possible that $f(4) = 6$.

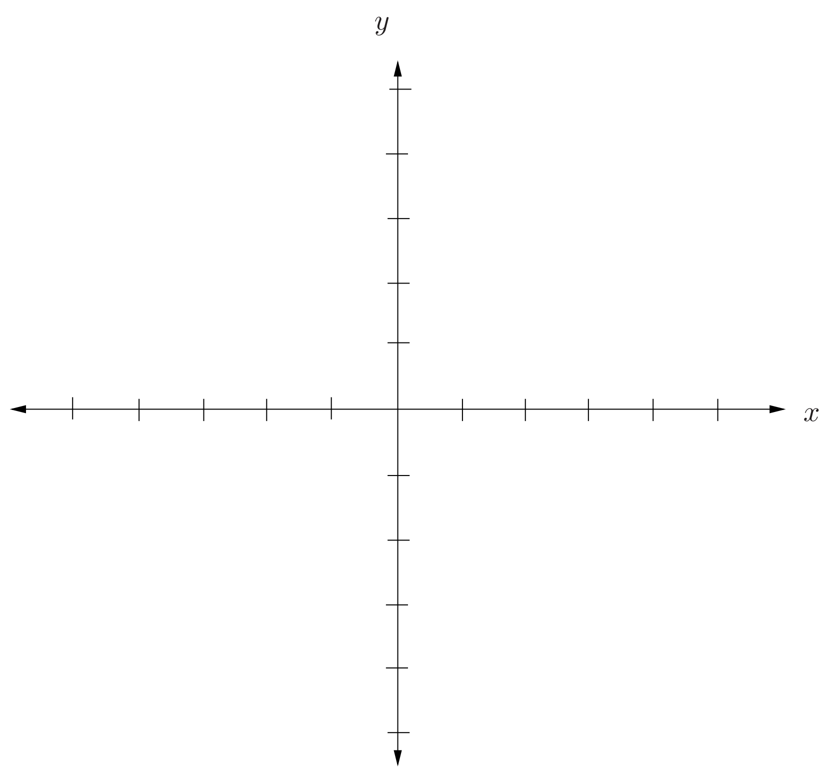
True False

(g) Suppose that $\lim_{x \to 2} f(x) = 4$. Then $f(1.999)$ is closer to 4 then $f(1.99)$ is.

True False

2.(points) On the axes below, sketch a graph of a single **function**, g, with **all** of the following properties.

- $g(-2) = g(2) = 1$
- $g'(x) = 0$ for $x < -2$ and $x > 2$
- $g'(x) < 0$ for $-2 < x < 2$
- $g''(x) > 0$ for $-2 < x < 0$
- $g''(x) < 0$ for $0 < x < 2$
- $\lim_{x \to -2^+} g(x) = \infty$ and $\lim_{x \to 2^-} g(x) = -\infty$



3. (points) A group of researchers in Costa Rica is studying the number of resplendent quetzals (these are birds) that nest in Monteverde Cloud Forest Preserve each year. The function $f(t)$ gives the number of quetzals the researchers count in the park on day t. The researchers determine that on day T the maximum number of quetzals are in the park. Write a transformation (?expression?) of $f(t)$ that models each situation.

(a) After determining the function $f(t)$, the researchers discover they forgot to factor in an area of the park that houses 50 quetzal year round.

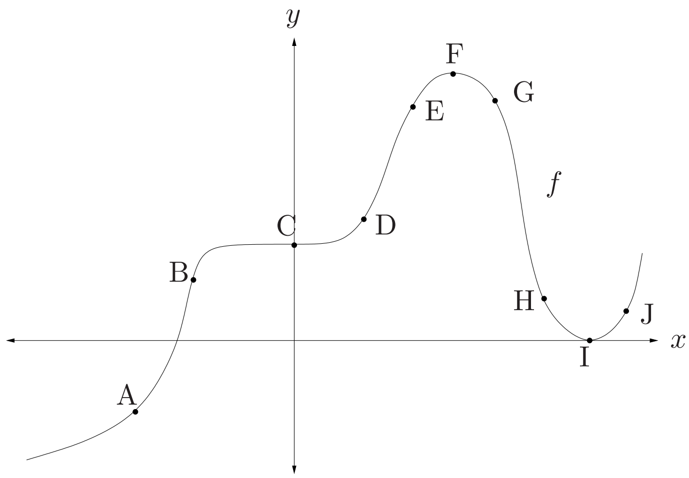
(b) The researchers discover another sloppy calculation that shows them that the maximum number of quetzal are actually in the park 5 days earlier then their original calculation.

(c) The number of visitors to the park is a function of the number of quetzals in the park. Suppose the function is given by $g(q)$ where q is the number of quetzal in the park. Write a function that determines the number of visitors to the park on day t.

4. (points) Consider a cylinder of radius r and height h. Find a formula for the volume of the cylinder in terms of the radius given that 6 times the height plus 2 times the radius must equal 36.

5. (points) The number of socks you own decreases exponentially according to the number of loads of laundry you've done since the beginning of the school year. After your first load of laundry you have 10 pairs of socks remaining and after 20 loads you are down to your last pair of socks. How many pairs of socks did you start with? [Show your work!]

6. (points) The graph of a continuous differentiable function f is given below.



(a) List all the points where f' and f'' are both positive or both negative.

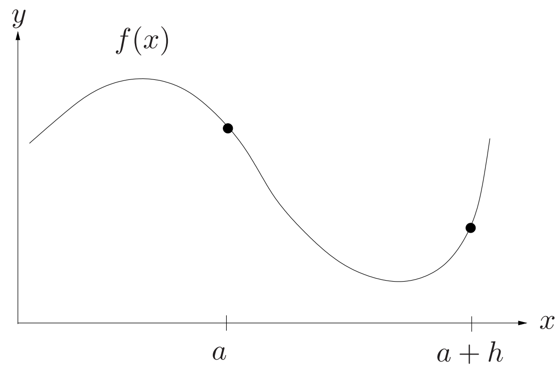
(b) List all the points where f and f' are both positive or both negative.

(c) List all the points where at least two of f, f', f'' are zero.

7. (points) For this problem f is differentiable everywhere.

(a) State the limit definition of the derivative for the function f at the point a.

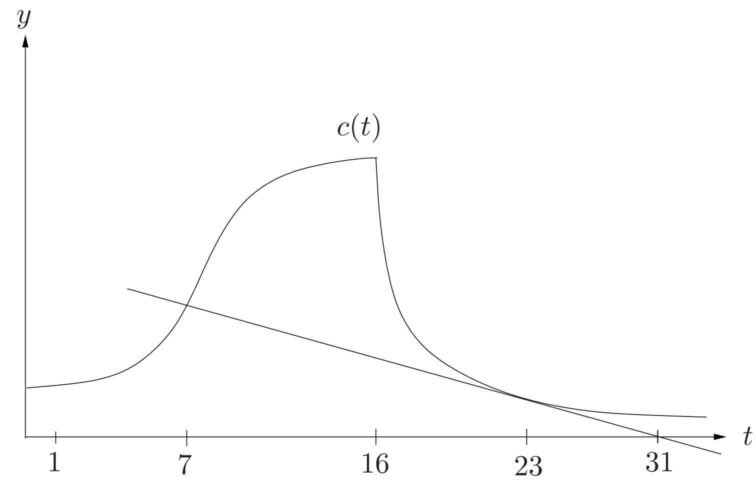
(b) Draw a picture that illustrates how the average rate of change of f is related to the derivative at the point a. Give a brief explanation of your illustration including how the limit as $h \to 0$ is demonstrated in your picture.



(c) Use the limit definition of the derivative to calculate $f'(2)$ when $f(x) = e^x$. Hint: It might be useful to use the fact that $\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$.

(d) Is it true that $\frac{d}{dx} f^{-1}(x) = (-1)f^{-2}(x)$? You may want to use part (c) and the fact that $\dfrac{d}{dx} \ln x = \dfrac{1}{x}$.

8. (points) As Sweetest Day (October 16^{th}) approaches, millions of Americans flock to stores to buy their special someone a card. The number of cards sold can be approximated by the continuous function $c(t)$ graphed below. $c(t)$ gives the number of cards sold (in tens of thousands) on day t where $t = 1$ corresponds to October 1.



(a) Are there any times t when $c(t)$ is not differentiable? If so, what are they?

(b) Explain the concavity of the graph between October 1 and October 16 in terms of the situation.

(c) If on October 1 there are 30,000 cards sold and on October 23 there are 25,000 cards sold, what is the average rate of change of $c(t)$ over this time?

(d) How many cards will be sold on October 7?

9. (points) As fall progresses the trees in the Arboretum gradually change color. The function $f(t)$ gives the percentage of leaves on a particular tree that have not yet begun to change colors as a function of the days since October 1st. (October 1st corresponds to $t = 0$!)

(a) Write a sentence in everyday terms describing what $f(10) = 15$ means.

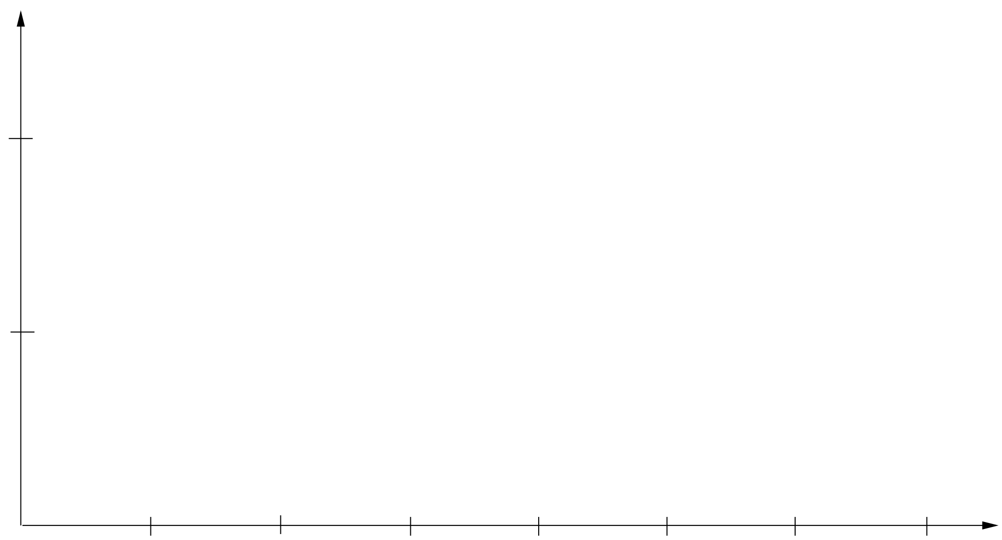
(b) Write a sentence in everyday terms describing what $f^{-1}(3) = 6$ means.

(c) You will go to photograph this tree on October 20, but only if over half of the leaves have begun to change color. Write an inequality describing the situation if you went and photographed the tree.

(d) Write a sentence in everyday terms describing what $f'(15) = 9$ means.

10. (points) The traffic on US-23 between Brighton and Ann Arbor is stop and go every weekday morning. I merge onto US-23 going south at Brighton at mile marker 58 travelling 35 miles per hour. The traffic is bad and I must immediately slow down, finally coming to a stop at mile marker 56. As soon as I stop I am able to speed up again, reaching a maximum speed of 70 miles per hour at mile marker 52. There are again traffic problems and I must slow again, coming to a stop at mile marker 48. Suppose my speed continues up and down periodically in the same pattern until I reach Ann Arbor at mile marker 45.

(a) Sketch a graph of my speed as I travel south on US-23 as a function of my position on US-23. Let mile marker 58 correspond to $d = 0$. Be sure to appropriately label the axes.



(b) Write a trigonometric equation describing my speed.

(c) What is my speed when I reach Ann Arbor?