MATH 108H — FINAL EXAM

April 26, 2010

NAME: ____

1. Do not open this exam until you are told to begin.
2. This exam has 16 pages including this cover. There are 8 problems, including one extra credit problem.
3. Write your name on the top of EVERY sheet of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use your calculator. However, you are NOT allowed to use it to evaluate integrals or take derivatives. If you use it in a significant way, explain how you used it. (For example, if you use it to graph arc tangent or something comparable to that.)
8. Turn **off** all cell phones.
9. I apologize for the less than exciting theme, but you guys didn't give me much time to rewrite it!

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	15	
4	10	
5	10	
6	15	
7	10	
8	12 (EC)	
TOTAL	100	

Name: ____

1. (5 points each) Evaluate the integrals. The fact that $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$ may be useful.

(a)

$$\int xe^x dx$$

(b)

$$\int_1^5 \frac{dx}{x\ln(x)}$$

Name: ____

(c)

$$\int \frac{x dx}{\sqrt{x^2+1}}$$

(d)

$$\int_0^1 \frac{dx}{\sqrt{x^2+16}}$$

Name: ____

2. (5 points each) Determine if the following converge or diverge. Be sure to justify your answer to receive any credit!

(a) The series:

$$\sum_{k=2}^{\infty} \frac{k^5}{2^k}$$

(b) The sequence:

$$a_n = \frac{\ln(n)}{n}$$

Name: ____

(c) The series:

$$\sum_{n=1}^{\infty} \frac{n^3}{n^3+1}$$

(d) The series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Name: ____

3. (5 points each) Consider the equation $r = 1 + 2\cos(\theta)$ given in polar coordinates.

(a) Sketch a graph of this equation for $0 \leq \theta \leq 2\pi$.

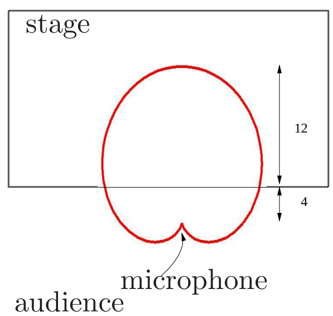
(b) Convert the equation to an equation in x, y-coordinates.

Name: ____

(c) Find the equation of the tangent line to the curve at $\theta = \frac{\pi}{4}$.

Name: ____

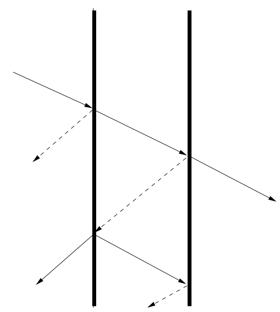
4. (10 points) After moving to South Carolina from Los Angeles, you decide to sample the southern lifestyle and attend a bbq where bluegrass music is played. This isn't any bluegrass band, they know that one should use a microphone with a cardioid pickup pattern because it suppresses noise from the audience. Suppose the microphone is placed 4 meters from the front of the stage and the boundary of the optimal pickup region is given by the cardioid $r = 8 + 8\sin\theta$, where r is measured in meters and the microphone is at the pole. The band wants to know the area they will have on stage within the optimal pickup range of the microphone. Being a good calculus student, answer the question for them.



Name: ____

Name: ____

5. (10 points) You get a glass of sweet tea. The glass is double-paned so that it consists of two parallel panes of glass with a small spacing between them. Suppose that each pane reflects a fraction p of the incoming light and transmits the remaining light. Considering all reflections of light between the panes, what fraction of incoming light is ultimately transmitted by the by the glass into your sweet tea? (Assume the amount of incoming light is 1.)



Name: ____

6. (5 points each) **(a)** Show that the derivative of $y = \mathrm{Sin}^{-1}(x)$ is $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. (Hint: Move the "sine" to the other side and use implicit differentiation.)

Name: ____

(b) Find $P_3(x)$ for $y = \text{Sin}^{-1}(x)$ around $a = 0$. Be sure to show all your work!

Name: ____

(c) At the bluegrass restaurant there is a trough of length L has a cross section in the shape of a semicircle with radius r meters. When the trough is filled with grits to a level h meters measured from the top of the trough, the volume of the grits is given by

$$V = L\left(\frac{\pi r^2}{2} - r^2\,\mathrm{Sin}^{-1}\left(\frac{h}{r}\right) - h\sqrt{r^2 - h^2}\right).$$

Show that if h is small in comparison to r, then

$$V \approx L\left(\frac{\pi r^2}{2} - 2rh + \frac{h^3}{2r}\right).$$

Name: ____

7. (10 points) The flux of a liquid flowing through a tube is the rate of flow, i.e., it is the volume per unit time that flows through. For instance, in a tube that has a circular cross section with radius R, if a fluid flows with a constant velocity then the flux is given by

$$\text{Flux} = v\pi R^2.$$

Note if the velocity is constant, the flux is just the velocity times the area.

As you shovel more and more fried food into yourself you begin to visualize your arteries gradually clogging. The law of laminar flow states that the velocity v of blood flowing through a blood vessel of radius R and length l at a distance of r from the central axis is given by

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the blood vessel and η is the viscosity of the blood. Assume that l, P, R, and η are constants. However, note that velocity is NOT constant in the tube, it flows faster in the center for instance! Show that the flux F is given by

$$F = \frac{\pi P R^4}{8\eta l}.$$

This is known as Poiseuille's Law.

Name: ____

Name: ____

8. (Extra Credit: 3 points each)
(a) The phenomenon that states that very small changes in initial conditions can cause large changes in long run behavior is often referred to as the ______________ effect.

(b) Edward Lorenz, one of the pioneers in studying Chaos theory, was a ______________ by trade.

(c) True/False: It is possible for chaos to arise in a situation modeled by a simple mathematical equation.

(d) Give an example of a fractal occurring in nature and explain why it is a fractal: