MATH 108H — FIRST MIDTERM EXAM

February 17, 2010

NAME: _____

1. Do not open this exam until you are told to begin.
2. This exam has 12 pages including this cover. There are 6 problems.
3. Write your name on the top of EVERY sheet of the exam! (Except the formula sheet.)
4. Do not separate the pages of the exam. (Except the formula sheet.)
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use your calculator. However, you are NOT allowed to use it to evaluate integrals or take derivatives. If you use it in a significant way, explain how you used it. (For example, if you use it to graph arc tangent or something comparable to that.)
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	15	
3	20	
4	20	
5	15	
6	10	
TOTAL	100	

Name: _____

1. (5 points each) Evaluate the following integrals:

(a)

$$\int_1^5 \frac{x}{1+x^2}\,dx$$

(b)

$$\int \sin^3(\theta)\cos^5(\theta)d\theta$$

Name: _____

(c)

$$\int y^4 \ln(y) dy$$

(d)

$$\int_{e^3}^{e^5} \frac{dx}{x\sqrt{\ln(x)^2 - 1}}$$

Name: _____

2. (15 points each) Consider the region between the curve $y = \sin(x)\sqrt{\cos(x)}$ and the x-axis from $x = 0$ to $x = \pi/2$. Find the volume of the region obtained by rotating this region around the line $y = -2$.

Name: _____

3. (10+5+5 points) A population of genetically engineered killer rabbits is placed on an island. Ten of these creatures are initially placed on the island. The time it takes for the population to reach N killer rabbits is given by

$$T(N) = \int_{10}^{N} \frac{15\,dx}{x(400-x)},$$

where T is measured in years after the killer rabbits where first placed on the island.

(a) Find a formula for $T(N)$ by evaluating the integral. Be sure to show all your work!

Name: _____

(b) How long does it take for there to be 200 killer rabbits roaming the island? Be sure to put your answer in a complete sentence and include units!

(c) Determine if the integral $T(400) = \int_{10}^{400} \frac{15\,dx}{x(400-x)}$ converges or diverges. What does your conclusion mean in terms of the killer rabbits on the island?

Name: _____

4. (10+5+5 points) The island containing the killer rabbits has only one convenient source of fresh water, a pond with radius 5 meters. The depth of the pond in meters at a distance of r meters from the center of the pond is given by $d(r) = 10\cos\left(\frac{\pi r}{10}\right)$ for $0 \leq r \leq 5$.

(a) Find the volume of the pond. Be sure to draw and label a typical slice!

Name: _____

(b) Unfortunately, the killer rabbits are smart too and begin to build a civilization. They use fresh water at an increasing rate of $2x$ m^3/day where x is measured in days. The pond is fed by a stream that flows into the pond at a rate of $\frac{50x^2}{x\sqrt{x^2+1}}$ m^3/day where again x is measured in days. Write an equation that gives the amount of fresh water in the pond after T days. It is ok if the equation contains derivatives or integrals, you do not need to evaluate them for this part. You can also assume the rabbits are the only animals taking water from the convenient source.

Name: _____

(b) When, if ever, do the rabbits completely drain the pond?

Name: _____

5. (15 points each) The rabbits have decided that they need a fitting monument to their glorious empire. They decide a pyramid would do nicely. After enslaving the native squirrel population, they force them to build the pyramid. The stone that will be used to build the pyramid has density 3000 kg/m^3. The pyramid will have a square base 100 m by 100 m and a height of 75 meters. Find the total amount of work the squirrels must do to build the pyramid. (Ignore any friction or other such issues.)

Name: _____

6. (10 points) Recall that if $y = f(x)$ is a continuous and differentiable function on an interval $[a, b]$, then the arc length of the curve from $x = a$ to $x = b$ is given by

$$\text{Arc Length} = \int_a^b \sqrt{1 + (f'(x))^2}dx.$$

Explain in as much detail as possible where this formula comes from. You should include pictures in your explanation.

Useful Formulas

$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \csc(x)dx = -\ln|\csc(x) + \cot(x)| + C$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}.$$