

# MATH 2060 — FIRST MIDTERM EXAM

February 12, 2014

NAME: \_\_\_\_\_

1. Do not open this exam until you are told to begin.
2. This exam has 12 pages including this cover. There are 5 problems.
3. Write your name on the top of EVERY sheet of the exam at the START of the exam!
4. Do not separate the pages of the exam.
5. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so I will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work for each exercise so that I can see not only the answer but also how you obtained it.
7. You may use a non-graphing calculator. You are NOT allowed to use it to do anything significant such as integrating, taking derivatives, etc.
8. Turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	15	
4	20	
5	25	
TOTAL	100	

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1. (4 points each) Calculate the following or state the quantity doesn't make sense. Let  $\mathbf{v} = 2\mathbf{i} + 3\mathbf{k}$  and  $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ .

(a)  $\mathbf{v} + \mathbf{w}$

(b)  $2\mathbf{v} \times \mathbf{w}$

(c)  $(\mathbf{v} \cdot \mathbf{w}) \times \mathbf{v}$

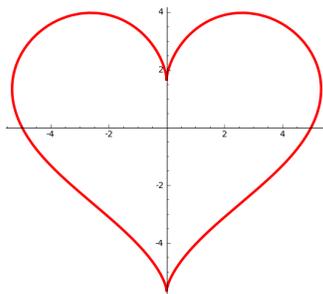
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(d) the angle between  $\mathbf{v}$  and  $\mathbf{w}$

(e) the projection of  $\mathbf{v}$  onto  $\mathbf{w}$

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2. (5 points each) You are determined to make this Valentine's day particularly special for your significant other. Knowing s/he loves chocolates, you begin by buying an expensive box of chocolates in a heart shaped box pictured here:



Unfortunately, when you arrive home and take it out of the bag you notice a bunch of ants trying to get into the box. They are walking along the outside of the box. At time  $t$  (measured in seconds) the lead ant's position (measured in cm) is given by the parametric equations

$$x(t) = \frac{16 \sin^3(t/3)}{3}$$

$$y(t) = \frac{1}{3}(13 \cos(t/3) - 5 \cos(2t/3) - 2 \cos(t) - \cos(4t/3)).$$

(a) Give a vector equation  $\mathbf{r}(t)$  that gives the lead ant's position at time  $t$ . What is the lead ant's position at time  $t = \pi$  seconds? Illustrate  $\mathbf{r}(\pi)$  on the graph above.

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(b) What is the lead ant's velocity at time  $t = \pi$  seconds? What is its speed?

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(c) What is the lead ant's acceleration at time  $t = \pi$  seconds?

(d) What is the curvature of the path followed by the lead ant at time  $t = \pi$  seconds?

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**3.** (15 points) Upon removing the ants from your box of chocolates you quickly stuff the box into your backpack and hop a flight to Central America. Your significant other is a huge orchid fan and you are going to retrieve a rare orchid from the jungle for a second present. You follow a path through the jungle given by the vector equation  $\mathbf{r}(t) = t\mathbf{i} + \sin(\pi t)\mathbf{j}$  measured in meters where  $t$  is given in minutes. Unbeknownst to you, a pair of chocolate hungry squirrel monkeys are tracking you. At  $t = 0$  the lead monkey is located in a tree at  $(-5, 5, 10)$ . The other monkey is located at  $(10, 2, 5)$ . Both monkeys take straight line paths at constant speeds. After two minutes the first monkey is at  $(-\frac{1}{2}, \frac{5}{2}, 5)$  and after one minute the second monkey is at  $(7, 1, \frac{5}{2})$ . The monkeys are particularly smart and have figured out your exact path. If they land on your path they will stay there and wait for you if you haven't arrived yet. If both monkeys encounter you on your path they will take your chocolates. Do you make it out with the chocolates? Be sure to fully justify your answer!!!

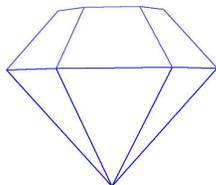
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4. (20 points) You've grabbed the orchid and are heading out of the jungle. Unfortunately more monkeys have gathered on the path you came in on and look like maybe they want more to eat than just chocolates now. You decide to take a different path out. This involves using a boat to cross a river filled with piranha. You start on the west bank of the river (the water flows north) and you paddle at 2 meters per second straight towards the other shore. The river flows north at 3 meters per second. The piranha jump out of the water at a 30 degree angle to the water heading directly northeast at 4 meters per second. The piranha aren't as smart as the monkeys so the only way they can land on your boat is if their speed in the direction traveled by the boat is greater than the speed of the boat. If they land on your boat you have to ditch the orchid to fight the piranha off. Do you make it across the river with the orchid? Be sure to fully justify your answer!!!

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5. (5 + 10 + 10 points) After your crazy adventure in Central America you return home. You decide that jewelry always works for your significant other so decide to purchase a nice diamond. The diamond you get is pictured here:



The bottom of the diamond has 6 sides. Choose coordinate axes so the bottom tip of the diamond is at  $(0, 0, 0)$ . The height of the bottom part of the diamond is twice the height of the top, so you can say the bottom has height 1 and the top has height  $1/2$ .

Your significant other obviously finds intelligence incredibly attractive, so it is likely s/he will be impressed with your ability to answer the following questions about the diamond.

(a) Give the coordinates of the three vertices of one of the bottom faces in the given coordinate system. (You may rotate the diamond so that one of the vertices is at a nice point like  $(1, 0, 1)$ . Now you only need to find one more vertex. Consider what this looks like viewed from above. You need six equally spaced points in the plane with one of them at  $(1, 0)$ .)

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(b) Find the equation of the plane containing the face from part (a).

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(c) Find the angle between the face from part (a) and the  $yz$ -plane.