Math 333 Problem Set 8
Due: 04/18/16

Be sure to list EVERYONE in the that you talk to about the homework!

Throughout this homework F denotes a field.

1. Let $D : \mathbb{R}[x] \to \mathbb{R}[x]$ be the derivative map. Is D a homomorphism of rings? An isomorphism? Be sure to justify your answer.

2. Let $a, b \in F$ with $a \neq b$. Prove that $\gcd(x - a, x - b) = 1_F$ in $F[x]$.

3. Modify the proof of the Euclidean algorithm we gave for $\mathbb{Z}$ to prove there is a Euclidean algorithm for $F[x]$. Use your algorithm to find the greatest common divisor of $f = 4x^4 + 2x^3 + 6x^2 + 4x + 5$ and $g = 3x^3 + 5x^2 + 6x$ in $(\mathbb{Z}/7\mathbb{Z})[x]$. Express $\gcd(f, g)$ as a linear combination of f and g.

4. Prove that $x^2 + 1$ is irreducible in $\mathbb{Q}[x]$.

5. List all associates of $x^2 + x + 1$ in $(\mathbb{Z}/5\mathbb{Z})[x]$.

6. Prove that $f \in F[x]$ is irreducible if and only if for every $g \in F[x]$, either $f \mid g$ or $\gcd(f, g) = 1_F$.

7. Find a nonzero polynomial in $(\mathbb{Z}/3\mathbb{Z})[x]$ that induces the zero function on $\mathbb{Z}/3\mathbb{Z}$.

8. Use the factor theorem to show that $x^7 - x$ factors in $(\mathbb{Z}/7\mathbb{Z})[x]$ as $x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$ without doing any polynomial multiplication.

9. For what values of k is $x - 2$ a factor of $x^4 - 5x^3 + 5x^2 + 3x + k$ in $\mathbb{Q}[x]$.

10. If f and g are associates in $F[x]$, show they have the same roots in F. If f and g have the same roots in F, are they necessarily associates? Be sure to justify your answer.