Math 333 Problem Set 9
Due: 05/04/16

Be sure to list EVERYONE in the that you talk to about the homework!

1. (a) Show that the set $\{(a, 0) : a \in \mathbb{Z}\}$ is an ideal in $\mathbb{Z} \times \mathbb{Z}$.
   (b) Show that the set $\{(a, a) : a \in \mathbb{Z}\}$ is not an ideal in $\mathbb{Z} \times \mathbb{Z}$.
2. Let R and S be rings and $I \subset R$, $J \subset S$ ideals. Show that $I \times J$ is an ideal in the ring $R \times S$.
3. Show that if I is an ideal in a field F, then $I = \langle 0_F \rangle$ or $I = F$.
4. List all the distinct principal ideals in $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$.
5. Let I be an ideal in R and S a subring of R. Prove that $I \cap S$ is an ideal in S.
6. (a) Let I and J be ideals in a ring R. Define $I + J = \{i + j : i \in I, j \in J\}$. Show this is an ideal in R that contains I and J.
   (b) Let $a, b \in \mathbb{Z}$ and set $d = \gcd(a, b)$. Show that $\langle a \rangle + \langle b \rangle = \langle d \rangle$.
7. Let F be a field. Show that every ideal in the ring $F[x]$ is principal.
8. (a) Prove that the set S of rational numbers (in lowest terms) with odd denominators is a subring of $\mathbb{Q}$.
   (b) Let I be the set of elements in S with even numerators. Prove that I is an ideal in S.
   (c) Show the set S/I consists of exactly two distinct cosets.